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On the locating chromatic number of infinite trees

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Abstract

The locating chromatic number of a graph is the smallest integer n such that there exists a proper n-coloring c for which every vertex has a unique vector of distances to the color classes defined by c. If no such integer n exists, the locating chromatic number is infinite. We explore properties of the locating chromatic number of infinite graphs, focusing on sufficient and necessary conditions for an infinite tree to have a finite locating chromatic number. We conjectured that an infinite tree has a finite locating chromatic number if and only if the tree has bounded degree and is prunable. We prove that the conditions in the conjecture are sufficient and provide evidence as to why they might be necessary. Lastly, we compare the performance of our locating coloring algorithm with that of known locating coloring algorithms. Our algorithm outperforms previously known algorithms for trees with many branches.

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1. Introduction

An *infinite graph* is a graph with an infinite number of vertices. We use the term graph to refer to both infinite and finite graphs. In infinite graphs, a path between two vertices is required to have

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a finite number of internal vertices, i.e., a finite path. Consequently, the distance between vertices connected by a path is finite. The notion of infinite distance in infinite graphs is used similarly to its definition in finite graphs, i.e., to describe two vertices that are not connected by a path. Infinite paths are either a ray (a path with one vertex of degree one and an infinite end) or a double ray (a path with no vertex of degree one and two infinite ends). As in finite graphs, an infinite graph is connected if there is a path between every pair of vertices.

Recall that the unique connected 2-regular graph on n vertices is a cycle. One can see that the double ray is the unique connected 2-regular graph with infinitely countable vertices. In some references, for example [7], the double ray is indeed considered an infinite cycle. This is further supported by the fact that a cycle on n vertices is the Cayley graph of the finite cyclic group of order n, that is $\mathbb{Z}_n$ ($C_n = \mathrm{Cay}(\mathbb{Z}_n, \{\pm 1\})$), and the double ray is the Cayley graph of the infinite cyclic group $\mathbb{Z}$, that is ($P_{2\infty} = \mathrm{Cay}(\mathbb{Z}, \{\pm 1\})$). On the other hand, some references do not consider the double ray as a cycle, see Chapter 8 in [8]. One reason is the unique path between every two vertices in a double ray, which is a property of a tree. Although the definition of an infinite cycle is not agreed upon, it does not affect the definition of an infinite tree. One can define an infinite tree to be a connected infinite graph without any finite cycles. One can also define infinite trees as infinite graphs with a unique path between any two vertices. This is also equivalent to the minimal connected infinite graphs.

The locating chromatic number of a connected graph is the smallest integer n such that there exists a proper n-coloring c for which every vertex has a unique vector of distances to the color classes defined by c. If no such integer exists, we say that the locating chromatic number of the graph is infinite. Note that each vertex is required to have a unique vector of distances. Since the distance between two vertices is an integer, the number of possible vectors of distances is countable. This means that a connected uncountable graph has infinite locating chromatic number. So we are left to consider countably infinite graphs, and thus we use the term infinite graph to refer to countably infinite graphs, unless stated otherwise.

In Section 2, we explore properties of the locating chromatic number of infinite graphs, with a focus on necessary and sufficient conditions for an infinite tree to have a finite locating chromatic number. We will use the characterization of infinite trees with finite metric dimension from [5], because having finite metric dimension is a sufficient condition for an infinite tree to have a finite locating chromatic number. The coloring algorithms in [4], [9], and [10] can be adjusted for infinite trees and are therefore useful for obtaining a sufficient condition for infinite trees to have a finite locating chromatic number. We also use the relation between the maximum degree of a graph and its locating chromatic number. This relation gives a necessary condition for an infinite tree to have a finite locating chromatic number, namely bounded degree.

In Section 3, we present our conjecture on a characterization of infinite trees with finite locating chromatic number. We conjecture that an infinite tree has a finite locating chromatic number if and only if it is prunable, that is, the infinite tree reduces to a single vertex after a finite sequence of removing all non-branch vertices from all end-paths. We prove that this condition is sufficient for an infinite tree to have a finite locating chromatic number, and we also provide evidence suggesting that this condition may also be necessary.

In the last section, we combine our previous locating coloring algorithm from [4] with the locating coloring algorithm used in this paper. We use a locating coloring to prove that our conjec-

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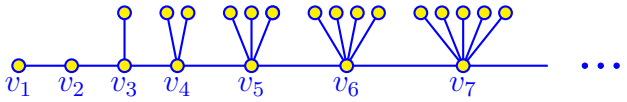


Figure 1: A graph with finite and unbounded degree.

ture provides a sufficient condition for an infinite tree to have a finite locating chromatic number. The new locating coloring works for all trees, finite or infinite. We analyze the performance of our algorithm compared to that of known locating coloring algorithms. Our algorithm outperforms previously known algorithms, especially for trees with many branches (see Table 1).

1.1. Preliminaries

Let $G = (V, E)$ be a simple connected graph. For an integer $k \geq 1$, a map $c : V(G) \to \{1, 2, \cdots k\}$ is called a *k-coloring* of G if $c(u) \neq c(v)$ for any two adjacent vertices $u, v \in V(G)$. The *chromatic number* of G, denoted by $\chi(G)$, is the smallest positive integer k such that G has a *k-coloring*. Let $\{X_1, X_2, \cdots, X_k\}$ be the color class of V induced by a k-coloring c. The *color code* $a_c(v)$ of a vertex v is the ordered $k-$tuple $(d(v, X_1), d(v, X_2), \cdots, d(v, X_k))$, where $d(v, X_i) = \min\{d(v, x) \mid x \in X_i\}$. If all vertices of G have distinct color codes, then c is called a *locating k-coloring* of G. The *locating-chromatic number* of G, denoted by $\chi_L(G)$, is the smallest positive integer k such that G has a locating k-coloring. If no integer k exists for which G has a locating k-coloring, then $\chi_L(G) = \infty$.

A graph is *locally finite* or has *finite degree* if the degree of every vertex is finite. We say that a graph G has *bounded degree* if there exists an integer M such that $\deg(v) \leq M$ for all $v \in V(G)$, which is equivalent to saying the maximum degree exists. Note that bounded degree implies finite degree but not conversely; see Figure 1 for an example of a locally finite graph with unbounded degree.

Adopting the notation in [5], a ray $P_{1\infty}$ and a double ray $P_{2\infty}$ are infinite paths at one end and both ends, respectively. Formally, a ray and a double ray are infinite graphs with vertex set $\mathbb{Z}^+$ and $\mathbb{Z}$ respectively, with two vertices u and v are adjacent if $|u - v| = 1$.

In a finite tree, an *end-path* is a path connecting a leaf (a vertex of degree one) to its nearest branch (a vertex of degree at least three). For infinite graphs, we generalize the definition of an end-path to also include a ray with exactly one branch as its starting vertex. If there is an end-path from a branch, we call it an *end-branch*. A *palm* is an end-branch together with all its end-paths.

A *pruning* is an operation on a tree by removing some (possibly infinite) vertices of degree one or two from a tree to obtain a sub-tree. After pruning, the graph is still required to be a tree, this means that we can only remove the end parts of some end-paths in a tree. A *maximal pruning* or *max pruning* is a pruning in which we remove as many vertices as possible. Consequently, in a maximal pruning of a tree with branches, all end-paths except the branch vertices are removed. Note that pruning a tree could give us many possible sub-trees, but maximal pruning a tree gives a unique sub-tree. We call a tree *prunable* if it becomes the trivial tree (a single vertex) after a finite number of prunings. To check whether a tree is prunable, it is sufficient to check whether it becomes the trivial tree after a finite number of maximal prunings. Consider the infinite comb tree in Figure 3. A maximal pruning of the infinite comb tree leaves us with a double ray, and

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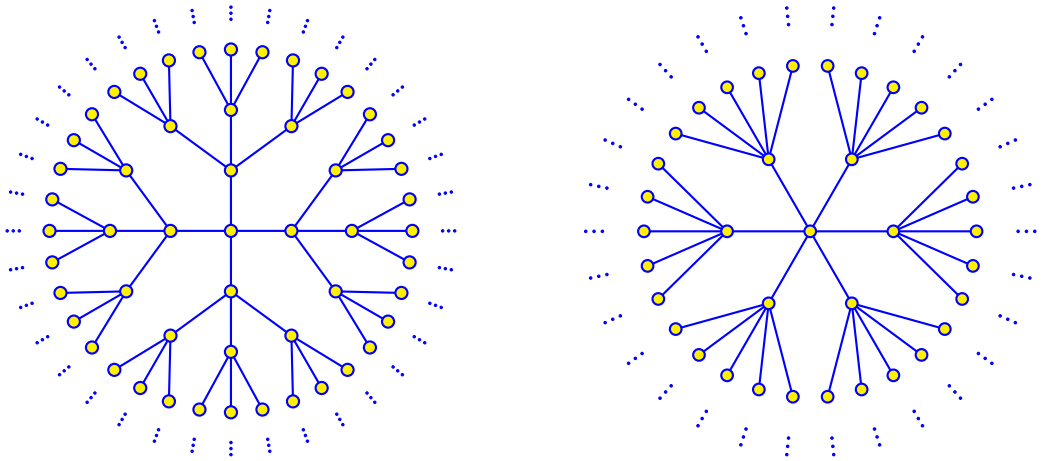


Figure 2: Infinite regular trees T_4 and T_6.

a maximal pruning of the double ray leaves us with a single vertex. Note that not all trees are prunable, two examples of unprunable trees are the infinite k-regular trees in Figure 2 and the trees in Figure 4.

2. Locating chromatic number of infinite graphs

In this section, we explore properties of the locating chromatic number of infinite graphs that are derived from known properties of the locating chromatic number of finite graphs. We focus primarily on properties that give necessary or sufficient conditions for an infinite tree to have a finite locating chromatic number. Although most of our discussion will focus on infinite trees, under certain conditions, the properties of the locating chromatic number of infinite trees also hold for infinite graphs in general. In such cases, we state the property for general infinite graphs.

We start this section by looking at the metric dimension of infinite trees in [5]. The metric dimension of a connected graph is the smallest number of vertices (called landmarks) such that every vertex has a unique vector of distances to the landmarks. This is a useful property to discuss because having a finite metric dimension is a sufficient condition for an infinite tree to have a finite locating chromatic number. The metric dimension of a finite tree is given below.

Theorem 2.1. *[11] Let T be a finite tree and b an end-branch in T. Let $EP(b)$ be the number of end-paths from b. The metric dimension of a tree T is*

$$\dim(T) = \sum_{b \text{ end-branch}} (EP(b) - 1)$$

where the sum is calculated over all end-branches of T.

The formula above has also been proven to work for infinite trees, see [5]. In the same paper, this formula is used as the baseline for characterizing infinite trees with finite metric dimension. We can see from the formula that the requirement for an infinite tree to have a finite metric dimension is having a finite number of end-branches, and each end-branch has a finite number of end-paths. This is equivalent to having a finite number of branches, and every vertex has a finite degree.

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Theorem 2.2. *[5] An infinite tree has finite metric dimension if and only if it is locally finite and has a finite number of branches.*

Although the exact formula for the locating chromatic number of finite trees is unknown, there are many known locating coloring algorithms that are tight for certain trees. These algorithms provide upper bounds for the locating chromatic number of trees. Such upper bounds are useful for determining sufficient conditions for an infinite tree to have a finite locating chromatic number. One well-known upper bound for the locating chromatic number of a finite graph G involves its chromatic number and metric dimension.

Theorem 2.3. *[6]* $\chi_L(G) \leq \chi(G) + \dim(G)$.

This bound is achieved by coloring all vertices of G by a $\chi(G)$-coloring and recoloring the landmarks of G with $\dim(G)$ unique additional colors. This coloring also works for infinite graphs, so we can use Theorem 2.3 to find necessary conditions for an infinite tree to have a finite locating chromatic number. The chromatic number of a tree is 2 as it is bipartite, so having finite metric dimension is sufficient for an infinite tree to have a finite locating chromatic number. This condition is further strengthened by the locating coloring algorithms for trees in [4], [9], and [10], with the following upper bounds.

Theorem 2.4. *For $k \geq 0$, let b_k be the number of branches in a finite tree T with at least i end-paths from that branch. For $i = 1, \ldots, b_2$, let P_i be the i^{th} branch with at least two end-path together with its end-paths. Let l_i be the number of leaves in P_i and p_i be the number of end-paths with length one in P_i. The following are three upper bounds of $\chi_L(T)$ from three different locating coloring algorithms.*

$$\textit{[4]} \quad \chi_L(T) \leq b_2 + \max\left\{ \left\lceil \sqrt{l_i} \right\rceil, p_i \,\middle|\, i = 1, \ldots, b_2 \right\}$$

$$\textit{[9]} \quad \chi_L(T) \leq l - b_1 + 2$$

$$\textit{[10]} \quad \chi_L(T) \leq 2 - 2b_2 + \sum_{i=1}^{b_2} \chi_L(P_i) \leq 2 + \sum_{i=1}^{b_2} (l_i - 1)$$

To use Theorem 2.4 for infinite trees, we need to include rays from branches as end paths. The upper bounds become the following.

Theorem 2.5. *Let T be a tree (finite or infinite) with notation as in Theorem 2.4. Let r and r_i be the number of rays in T and P_i that contain exactly one branch as the starting vertex, respectively.*

$$\chi_L(T) \leq b_2 + \max\left\{ \left\lceil \sqrt{l_i + r_i} \right\rceil, p_i \,\middle|\, i = 1, \ldots, b_2 \right\} \tag{1}$$

$$\chi_L(T) \leq l + r - b_1 + 2 \tag{2}$$

$$\chi_L(T) \leq 2 - 2b_2 + \sum_{i=1}^{b_2} \chi_L(P_i) \leq 2 + \sum_{i=1}^{b_2} (l_i + r_i - 1) \tag{3}$$

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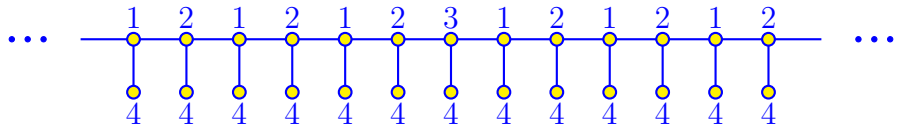


Figure 3: A locating 4-coloring of the infinite comb graph.

Compare to Theorem 2.3, the upper bounds (1) and (3) in Theorem 2.5 no longer require the number of end-branches to be finite, but only the number of end-branches with at least two end-paths to be finite. However, these two conditions are equivalent to having a finite number of branches. So, from known locating coloring algorithms, the sufficient condition for an infinite tree to have a finite locating chromatic number is still locally finite and has a finite number of branches. This sufficient condition is not a necessary condition. See the infinite comb graph in Figure 3. This graph has infinite branches and a locating 4-coloring.

One necessary condition for an infinite graph to have a finite locating chromatic number is related to its maximum degree. For finite graphs, the relation between the locating chromatic number of a graph and its maximum degree is given below.

Theorem 2.6. *[10] Let G be a finite graph with maximum degree $\Delta \geq 3$, then*

$$\chi_L(G) \geq 3 + \left\lceil \log_3 \left(\frac{\Delta}{4} \right) \right\rceil .$$

The proof of Theorem 2.6 in [10] looks at a local property of the graph. This property still works for infinite graphs, and hence having finite locating chromatic number implies having finite maximum degree.

Theorem 2.7. *If G is an infinite graph with a finite locating chromatic number, then G has bounded degree.*

3. Conjecture on infinite trees with finite locating chromatic number

Recall the definition of pruning a tree as removing some vertices of degree one or two from a tree to obtain a subtree. A tree is prunable if it becomes the trivial tree (a single vertex) after a finite number of prunings. We have established in Theorem 2.7 that having a bounded degree is a necessary condition for an infinite graph to have a finite locating chromatic number. We conjecture that these two conditions are the necessary and sufficient conditions for an infinite tree to have a finite locating chromatic number.

Conjecture 1. *An infinite tree T has a finite locating chromatic number if and only if T has bounded degree and is prunable.*

We have been able to prove that the conditions in Conjecture 1 are sufficient conditions for a tree to have finite locating chromatic number. For now, we are unable to prove that it is also a necessary condition. However, we will provide some supporting evidence on why we believe it is.

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In order for the conditions in Conjecture 1 to be a sufficient condition, we need to prove that if a tree satisfies the condition, then it has finite locating chromatic number. We will prove that the condition in this conjecture is a sufficient condition by repeatedly applying the following theorem.

Theorem 3.1. *Let T be an infinite tree with bounded degree and $MP(T)$ be the tree obtained by maximal pruning of T. Suppose that $\chi_L(MP(T))$ is finite, then $\chi_L(T)$ is finite.*

Proof of Theorem 3.1. Let $\chi_L(MP(T)) = m$ for some positive integer m and let c be a locating m-coloring of $MP(T)$. We will construct c', a locating $(m + \Delta)$-coloring of T. Consider the following coloring algorithm.

1. For every end-branch of T, assign an ordering of all end-paths starting from the end-branch.
2. For every vertex v of T;
   (a) If v is in $MP(T)$, then $c'(v) = c(v)$;
   (b) If v is not in $MP(T)$, then let b be the nearest end-branch from v;
   (c) If there are more than one end-paths from b, suppose that v is located in the i^{th} end-path from b, then

$$c'(v) = \begin{cases} c(b), & d(b,v) \text{ is even,} \\ m+i, & d(b,v) \text{ is odd.} \end{cases}.$$

   (d) If there is only one end-path from b, then

$$c'(v) = \begin{cases} m+2, & d(b,v) \text{ is even,} \\ m+1, & d(b,v) \text{ is odd.} \end{cases}.$$

Since T has bounded degree, all the vertices are colored by no more than $m + \Delta$ colors. Now, we prove that c' is a locating coloring of T. Consider two vertices having the same distances to the colors $m+1, m+2, \ldots$, and $m+\Delta$. These two vertices must be in one of these two conditions: (i) both in $MP(T)$ or (ii) both in the i^{th} end-paths from different end-branches with the same distance to the end-branch. If these two vertices satisfy the first condition, then they will be distinguished by the color class that distinguishes them in c. If the second condition is satisfied, then they are distinguished by the color class that distinguishes their nearest end-branches. Therefore, c' is a locating coloring. □

Now, we will provide evidence to support the necessary part of the conjecture. The contrapositive of the forward direction states that

$$MP^n(T) \neq K_1, \forall n \in \mathbb{Z}^+ \quad \Longrightarrow \quad \chi_L(T) = \infty, \tag{4}$$

where the power means compositions. We will provide some examples that support this statement. We consider two separate conditions for $MP^n(T) \neq K_1$ for all n,

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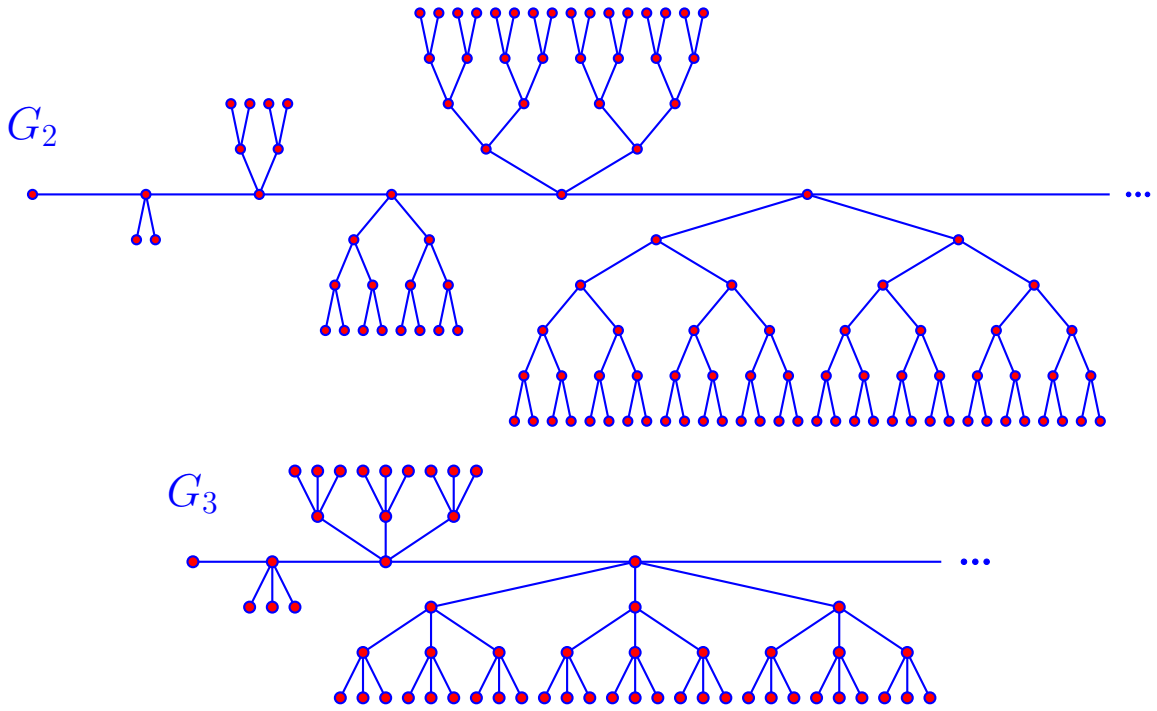


Figure 4: Graphs G_2 and G_3

(i) T has no end-path (graph T_k in Figure 2); and

(ii) $MP^k(T) = T$ for some k (graph G_n in Figure 4).

Theorem 3.2. *For $k \geq 3$, let T_k be the infinite k-regular tree (see Figure 2), then $\chi_L(T_k) = \infty$.*

Proof. By a contradiction, suppose there was an integer $k \geq 3$ with $\chi_L(T_k) = t$. Let c be a locating t-coloring of T_k and let $v \in T$ with $a_c(v) = (a_1, a_2, \cdots, a_t)$. Let $D_n(v)$ be the set of all vertices at distance n from v, then $|D_n(v)| = k(k-1)^{n-1}$. Since $n^\alpha = o(\beta^n)$ for every real number $\alpha, \beta > 1$, there is an integer N such that for every $n \geq N$, $(2n)^t < k(k-1)^{n-1}$.

Let $n = \max(a_1, a_2, \cdots, a_t, N)$. Note that the distance from a vertex $u \in D_n(v)$ to any color i is no more than $2a_i \leq 2n$, and hence a vertex in $D_n(v)$ has at most $(2n)^t$ possible color codes. Since $D_n(T)$ has $k(k-1)^{n-1}$ vertices, then there are at least two vertices with the same color code, a contradiction. □

Let $T(n,k)$ be the complete k-level n-ary tree ([13]) and let $V(P_{1\infty}) = \{v_0, v_1, v_2, \dots\}$. Let G_n be the graph obtained from $P_{1\infty}$ by attaching the center of $T(n,i)$ to v_i, see Figure 4.

Theorem 3.3. *For $n \geq 2$, $\chi_L(G_n) = \infty$.*

Proof. Suppose that $\chi_L(G_n) = t < \infty$ for some n. Since $(2i)^t = o(n^i)$, there is an integer K such that for $i \geq K$, $(2i)^t < n^i$. Note that two leaves in $T(n,i)$ can be only distinguished by the color of a vertex in $T(n,i)$, as they have the same distance to any vertex outside of $T(n,i)$. Since

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the diameter of $T(n,i)$ is $2i$, then there are at most $(2i)^t$ distinguished leaves in $T(n,i)$. Because the number of leaves in $T(n,i)$ is n^i and $(2i)^t < n^i$ for $i \geq K$, there are at least two leaves not distinguished in $T(n,i)$, a contradiction. □

4. Locating coloring algorithm

As a direct consequence of the proof of Theorem 1, we have the following corollary.

Corollary 4.1. *Let T be a tree with maximum degree Δ, then $\chi_L(T) \leq \chi_L(MP(T)) + \Delta$.*

To get a better upper bound for $\chi_L(T)$ for any tree, we combine our locating coloring algorithm with the locating coloring algorithm in [4]. Let T be a bounded degree tree with palms $P_1, P_2, \cdots$. Let u_i be the branch vertex in P_i. Let l_i be the number of end-paths in P_i and p_i be the number of such end-paths with length one. Let $Q_{i,j}$ be the j^{th} end-path in P_i.

Theorem 4.1. *Let T be a bounded degree tree, then $\chi_L(T) \leq \chi_L(MP(T)) + \max\left\{p_i, \left\lceil\sqrt{l_i}\right\rceil\right\}$.*

To prove Theorem 4.1, we utilize the following coloring algorithm.

Algorithm 1 Locating coloring of a tree T.

procedure A LOCATING COLORING OF A TREE T
Input: Tree T, locating coloring of $MP(T)$
Output: A locating coloring c of T
Color every vertex in $MP(T)$ with the existing locating coloring.
$n \leftarrow \max\{c(v) \mid v \in MP(T)\}$
$m \leftarrow \max\{p_i, \lceil\sqrt{l_i}\rceil\}$
Let $A = \{(a,b) \mid n+1 \leq a, b \leq n+m, a \neq b\}$
for $i = 1, 2, \cdots$ **do**
  Sort all end-paths in P_i in ascending order
  for $j = 1, 2, \cdots, l_i$ **do**
    if $j \leq m$ **then**
      Color $Q_{i,j}$ using $n+j$ and $c(u_i)$ alternately
    else
      $k \leftarrow j - m$
      Let (a_k, b_k) be the k^{th} element in A
      Color $Q_{i,j}$ using a_k and b_k alternately

Proof of Theorem 4.1. Let T be any tree. Color the tree T using Algorithm 1. We will prove that the coloring in Algorithm 1 is a locating coloring. Note that every vertex in P_i is colored by one of $\{c(u_i), n+1, n+2, \cdots, n+m\}$, so $d_T(v, c_i) = d_{MP(T)}(v, c_i)$ for every vertex v in $MP(T)$. Any two vertices in $MP(T)$ is distinguished by the locating coloring of $MP(T)$. A vertex in $MP(T)$ and a vertex in $T - MP(T)$ are distinguished by colors $\{n+1, n+2, \cdots, n+m\}$.

Suppose there were two vertices v and w with the same color code, then v and w must be in different palms P_i and P_j with $d(v,u_i) = d(w,u_j)$ and $c(u_i) = c(u_j)$. Since $MP(T)$ is colored

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by a locating coloring, then the color that distinguishes u_i and u_j will also distinguish v and w, a contradiction. □

Although Theorem 4.1 and Algorithm 1 give a good approximation for the locating chromatic number of a tree, it is not practical to use the theorem and the algorithm because we need the locating chromatic number of $MP(T)$ and its locating coloring. If T is a finite tree, then it is guaranteed that $MP^n(T)$ is a path P for some n. Since the locating chromatic number of a path is $\min\{3, |V(P)|\}$, we could apply Theorem 4.1 and Algorithm 1 repeatedly to get a more practical approximation of $\chi_L(T)$ and its locating coloring algorithm. This observation also emphasizes the importance of Conjecture 1, if the conjecture is true, we have a locating coloring of any infinite tree with a finite locating chromatic number.

Let T be a bounded degree tree (finite or infinite) such that $MP^n(T)$ is a path. For $i = 0, 1, \cdots, n$, define $T_i := MP^i(T)$, $l^{(i)}_{max}$ the maximum number of end-paths from a palm in T_i, and $p^{(i)}_{max}$ the maximum number of end-paths of length one from a palm in T_i.

Theorem 4.2. *Let T be a bounded degree tree such that $MP^n(T)$ is a path P, then*

$$\chi_L(T) \leq \min\{3, |V(P)|\} + \sum_{i=0}^{n-1} \max\left\{p^{(i)}_{max}, \left\lceil \sqrt{l^{(i)}_{max}} \right\rceil\right\}.$$

Proof. Use Theorem 4.1 repeatedly. □

We compare our locating coloring algorithm with some known locating coloring algorithms (see Table 1). For larger graphs with many branches, Theorem 4.2 performs much better than FM[9] and BP[4]. For small graphs, Theorem 4.2 is not always better than BP[4]. The case where BP[4] outperforms Theorem 4.2 occurs when BP[4] achieves the exact value, and in such cases Theorem 4.2 yields the same order of the locating chromatic number.

Table 1: Comparison of the locating-chromatic number of trees

Graph T	$\chi_{\mathbf{L}}(T)$			
	Exact value	**Computational value**		
		FM [9]	**BP** [4]	**Thm 4.2**
Amalgamation of star	$m+a$	$km-2k+2$	$k+m-1$	$k+m$
Banana tree	$k-1$	$n(k-3)+2$	$n+k-1$	$\lceil\sqrt{n}\,\rceil+k$
Caterpillar	$n+2$	$nm-m+2$	$n+m$	$n+3$
Complete n-ary tree	$(n+k-1)$	$n^k-n^{k-1}+2$	$n^{k-1}+n$	$nk+1$
Double star	$b+1$	$b+a$	$b+2$	$b+2$
Lobster	$n+2$	mn^2-mn+2	$n(m+1)$	$2n+3$
Firecracker-1	k	$n(k-3)+2$	$n+k-2$	$k+3$
Firecracker-2	$k-1$	$n(k-3)+2$	$n+k-2$	$k+3$
Olive tree	$\lceil\log_3\left(\frac{n}{4}\right)\rceil+3$	$k+1$	$\lceil\sqrt{k}\rceil+1$	$\lceil\sqrt{k}\rceil+1$
Star	n	n	n	n

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Note:

$\chi_L(T)$	Locating-chromatic number of T
Amalgamation of star	$S_{k,m}$; $H(a-1) < k \leq H(a), a \geq 1$ [2]
Banana tree	$B_{n,k}$; $k \geq 4,\ 1 < n \leq k-1$ [1]
Caterpillar	$C(m,n)$; $n, m \geq 3$ [3]
Complete n-ary tree	$T(n,k)$; $n \geq 2$; $k = 2, 3$ [13]
Double star	$S(a,b)$; $1 \leq a \leq b$, $b \geq 2$ [6]
Lobster	$Lb(m,n)$; $2 \leq m \leq 3(n+2)+1$, $n \geq 2$ [12]
Firecracker-1	$F_{n,k}$; $k \geq 5$, $2 \leq n \leq k-1$ [3]
Firecracker-2	$F_{n,k}$; $k \geq 5$, $n > k-1$ [3]
Olive tree	O_k; $k \geq 3$ [10]
Star	S_n [6]

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