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On the relations among edge magic total, edge antimagic total, and ASD-antimagic graphs

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Abstract

Let G be a simple and finite graph of order p and size q. The graph G is said to be edge magic total, for short EMT, if there is a bijection $\lambda : V(G) \cup E(G) \to \{1, 2, \ldots, p+q\}$ such that all edge sums $\lambda(x) + \lambda(xy) + \lambda(y)$, $xy \in E(G)$, are the same. If all edge sums are pairwise distinct, then G is called edge antimagic total (EAT). Let t be a positive integer that satisfies $\binom{t+1}{2} \leq q < \binom{t+2}{2}$. The graph G is said to have an ascending subgraph decomposition (ASD) if G can be decomposed into t subgraphs $H_1, H_2, \ldots, H_t$ without isolated vertices such that H_i is isomorphic to a proper subgraph of H_{i+1} for $1 \leq i \leq t-1$. A graph that admits an ascending subgraph decomposition is called an ASD graph. An ASD graph G is said to be ASD-antimagic if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, p+q\}$ such that all subgraph weights $w(H_i) = \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e), 1 \leq i \leq t$, are distinct. In this paper, we provide constructions of ASD-antimagic graphs arising from EMT or EAT graphs.

Keywords: ascending subgraph decomposition (ASD), ASD-antimagic labeling,
edge magic total labeling, edge antimagic total labeling
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1. Introduction

Throughout this paper, $G = (V, E)$ denotes a finite simple graph with nonempty vertex set $V(G)$ and nonempty edge set $E(G)$. In 1970, Kotzig and Rosa [14] introduced the notion of a magic total labeling as follows. A bijection $\lambda : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called an *edge magic total labeling (EMTL)* of G if there exists a constant k such that for any edge xy of G, the edge sum of xy, also called the edge weight, is

$$\lambda(x) + \lambda(xy) + \lambda(y) = k.$$

If all the edge sums are distinct, then λ is called an *edge antimagic total labeling (EATL)* of G. Furthermore, if the edge sums form an arithmetic sequence with the smallest sum a and difference d, then λ is called an (a, d)*-edge antimagic total labeling* ((a, d)*-EATL)* of G, see [19]. Moreover, if $\lambda(V(G)) = \{1, 2, \ldots, |V(G)|\}$, then λ is called a *super EMTL* or a *super EATL*. Figure 1 shows a super EMTL and a super EATL of the cycle C_5.

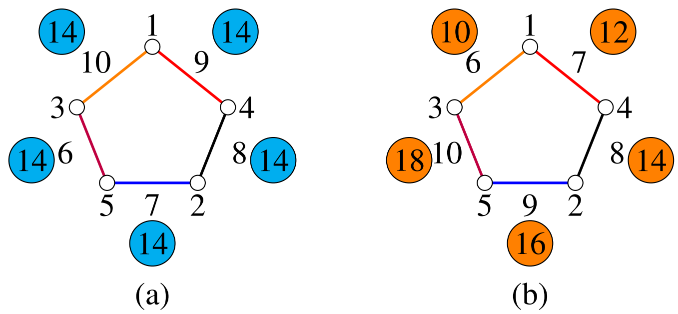

Figure 1. (a) A super EMTL of C_5. (b) A super $(10, 2)$-EATL of C_5.

In the last three years, many studies have been conducted on EMTL, EATL, and their applications. The results include the construction of super EMTL [7], the super EMTL of graphs with certain degree sequences [10], and the application of EMTL in machine learning [8] and RSA algorithms [12], while the application of EATL for RSA algorithms has been discussed in [13].

Since an edge can be seen as a K_2 subgraph, the notion of EMTL and EATL can then be generalized by considering the sum of labels in any subgraph. Let t be a positive integer and $\mathcal{H} = \{H_i \subseteq G, i = 1, 2, \ldots, t\}$ be a collection of t subgraphs of the graph G, where all H_i's are isomorphic to a graph H. The graph G is said to admit an H-decomposition if every edge in $E(G)$ belongs to exactly one H_i in $\mathcal{H}$, see [9].

For a graph G that admits an H-decomposition, a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called an H*-magic total labeling of* G if the *weight* of every subgraph H_i, denoted as $w(H_i)$, is a constant, where $w(H_i)$ is the sum of all vertex and edge labels in H_i, $i = 1, 2, \ldots, t$. If all weights are distinct, then the labeling f is called an H*-antimagic total labeling* of G. Furthermore, if the weights form an arithmetic progression starting from a with difference d, then f is called an (a, d)-H*-antimagic total labeling* of G, see [11]. Further, we call *an* H*-(anti)magic graph* for a graph admitting H-(anti)magic total labeling.

Many studies have been conducted on the H-(anti)magic total labelings and their applications. The results include the H-antimagic total labeling of the comb product graphs [1], the construction of (a, d)-H-antimagic total labelings using the EATL of smaller graphs [6], forbidden subgraphs of H-magic graphs [16], and the application of H-magic total labeling in combinatorial design [20].

Another way of decomposing a graph was introduced by Alavi et al. in [2], where all subgraphs are not isomorphic to each other. Let G be a graph of size q that satisfies $\binom{t+1}{2} \leq q < \binom{t+2}{2}$. The graph G is said to have an *ascending subgraph decomposition (ASD)* if G can be decomposed into t subgraphs $H_1, H_2, \ldots, H_t$, all without isolated vertices, such that H_i is isomorphic to a proper subgraph of H_{i+1}, for $1 \leq i \leq t-1$. In this case, we say that the graph is an *ascending subgraph decomposable (ASD)* graph. Hence, there exists a unique t for each q, and so the number of ascending subgraphs is restricted. However, there may be more than one way to decompose the graph. In the same article, the following conjecture was proposed.

Conjecture 1.1. [2] *Every graph of positive size has an ascending subgraph decomposition.*

There are many families of graphs that have already been proven to be ASD (see [15]), but, in general, Conjecture 1.1 remains open.

Motivated by the concept of H-magic and H-antimagic total labelings where all weights are counted in subgraphs that are isomorphic to each other, this paper deals with the existence of magic and antimagic labelings on ASD graphs, where a pair of subgraphs is not isomorphic. Recently, ASD-based magic and antimagic labelings were introduced by Pancahayani, Simanjuntak, and Uttunggadewa [17, 18]. The formal definitions for such labelings can be seen below.

Definition 1.1. [17] *Let G be an ASD graph and $\mathcal{H} = \{H_1, H_2, \ldots, H_t\}$ be a collection of t ascending subgraphs of G. Let $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ be a bijection of G. If there exists a constant k such that $w(H_i) = k$, for every $i \in \{1, 2, \ldots, t\}$, then f is called an **ASD-magic labeling** of G.*

Definition 1.2. [18] *Let G be an ASD graph and $\mathcal{H} = \{H_1, H_2, \ldots, H_t\}$ be a collection of t ascending subgraphs of G. Let $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ be a bijection on G. If there exist $\mathcal{H}$ and f such that $w(H_i) \neq w(H_j)$ for every $1 \leq i < j \leq t$, then G is called to admit an **ASD-antimagic labeling**. If the vertices are labeled with the smallest numbers, i.e., $f(V(G)) = \{1, 2, \ldots, |V(G)|\}$, then G is said to admit a **super ASD-antimagic labeling**. If f is a super ASD-antimagic labeling and all weights form an arithmetics progression with the first term a and difference d, then f is called a **super (a, d)-ASD-antimagic labeling**.*

Pancahayani, Simanjuntak, and Uttunggadewa [17] characterized some ASD-magic graphs such as stars, paths, and cycles. Pancahayani, Simanjuntak, and Uttunggadewa [18] also obtained an upper bound for d in (a, d)-ASD-antimagic graphs and constructed (a, d)-ASD-antimagic labelings for some product graphs. In Figure 2, examples of ASD-magic and super ASD-antimagic graphs are presented.

In this paper, we construct ASD-antimagic graphs from a "connector" graph that admits an edge magic total labeling (Section 2). We conclude by showing that some super (a, d)-edge antimagic total labelings of stars, paths, and cycles are also super ASD-antimagic labelings (Section 3).

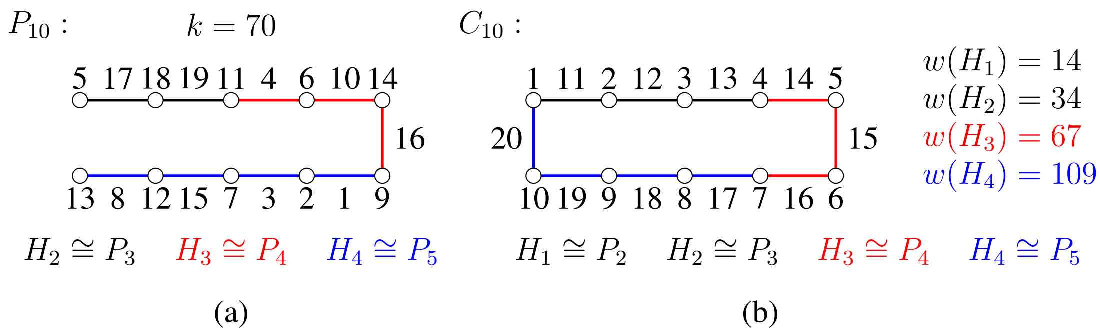

Figure 2. (a) An ASD-magic labeling on P_{10}. (b) A super ASD-antimagic labeling on C_{10}.

2. Constructions based on an EMT graph

In this section, we construct ASD-antimagic graphs from a special EMT graph called a *connector*. The construction involves a subdivision of an edge, defined as follows. A subdivision of an edge uv in G is the insertion of new k vertices $w_1, w_2, \ldots, w_k$ in uv accompanied by the joining of the original edge endpoints with the new vertices to form new edges $uw_1, w_1w_2, \ldots, w_{k-1}w_k, w_kv$.

Definition 2.1. *Let G^* be a connector, with $E(G^*) = \{e_1, e_2, \ldots, e_t\}$. Define $\mathcal{G}$ as a family of graphs constructed from G^* by substituting some edges e_i with subgraphs $G^*[e_i]$ obtained from at least one of the following procedures:*

A) subdivision of e_i, or

B) identifying one end vertex of e_i with a certain vertex of a connected graph,

in such a way that the following hold

i) for every i, $i = 1, 2, \ldots, t$, either $|V(G^*[e_i])| + |E(G^*[e_i])| = 2i + 1$ or $|V(G^*[e_i])| + |E(G^*[e_i])| = 2i + 3$, and

ii) $G^*[e_i]$ is isomorphic to a proper subgraph of $G^*[e_j]$, for $1 \le i < j \le t$.

From Definition 2.1 it is obvious that for each G^*, the corresponding $\mathcal{G}$ is an infinite family of graphs. Moreover, the following observation holds.

Observation 2.1. *Every graph G in $\mathcal{G}$ is ASD.*

Figure 3 shows an example of a connector graph G^* that is isomorphic to a cycle C_3 and three ASD graphs in the corresponding $\mathcal{G}$. Note that G_1 and G_2 are ASD graphs, where $|V(G^*[e_i])| + |E(G^*[e_i])| = 2i + 1$ for $i = 1, 2, 3$. The graph G_3 is an ASD graph, where $|V(G^*[e_i])| + |E(G^*[e_i])| = 2i + 3$ for each $i = 1, 2, 3$.

It turns out that all graphs in $\mathcal{G}$ are not only ASD, but also ASD-antimagic when the corresponding connector G^* is EMT. To simplify the notation, denote the sum of the order of a graph G and its size by $\sigma(G)$, that is $\sigma(G) = |V(G)| + |E(G)|$.

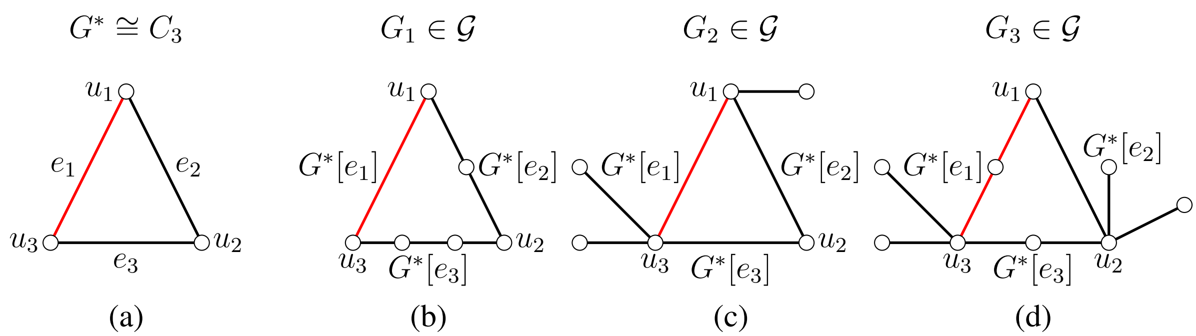

Figure 3. (a) A connector graph G^* isomorphic to a cycle C_3. (b) The graph G_1 derived from G^* by subdividing e_2 and e_3. (c) The graph G_2 derived from G^* by identifying u_1 with P_2 and u_3 with $K_{1,2}$. (d) The graph G_3 derived from G^* by subdividing e_1, identifying u_2 with $K_{1,2}$, subdividing e_3, and identifying e_3 with $K_{1,2}$.

Theorem 2.1. *If G^* is EMT, then every G in $\mathcal{G}$ is ASD-antimagic.*

Proof. Let f be an EMTL of G^*.

For the case when $\sigma(G^*[e_i]) = 2i+1$ for every $i \geq 1$, the label set for $G^*[e_1]$ is $\{f(e_1), f(u_1), f(v_1)\}$, and for $i \geq 2$, the label set for $G^*[e_i]$ is $\{f(e_i), f(u_i), f(v_i)\} \cup \{\sigma(G^*) + \sum_{j=1}^{i-1} [\sigma(G^*[e_j]) - 3] + m \mid 1 \leq m \leq \sigma(G^*[e_i]) - 3\}$.

For the case when $\sigma(G^*[e_i]) = 2i+3$ for every $i \geq 1$, the label set for $G^*[e_1]$ is $\{f(e_1), f(u_1), f(v_1)\} \cup \{\sigma(G^*) + m \mid 1 \leq m \leq \sigma(G^*[e_1]) - 3\}$, and for $i \geq 2$, the label set for $G^*[e_i]$ is $\{f(e_i), f(u_i), f(v_i)\} \cup \{\sigma(G^*) + \sum_{j=1}^{i-1} [\sigma(G^*[e_j]) - 3] + m \mid 1 \leq m \leq \sigma(G^*[e_i]) - 3\}$. □

Figure 4 provides labelings for the graphs in Figure 3 such that the connector graphs $G^* \cong C_3$ is EMT, and G_1, G_2, G_3 are ASD-antimagic.

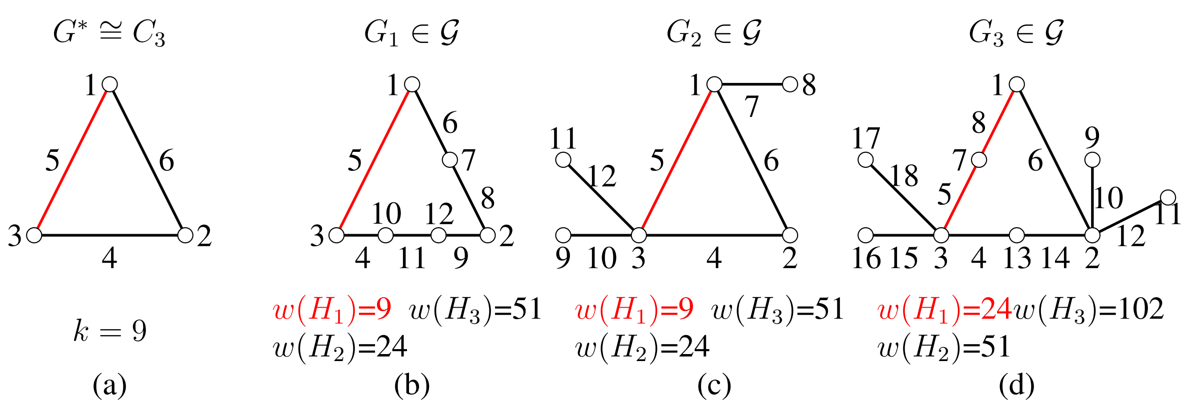

Figure 4. (a) An EMTL on G^*, (b) An ASD-antimagic labeling on G_1, (c) An ASD-antimagic labeling on G_2, (d) An ASD-antimagic labeling on G_3.

3. Constructions based on a super (a, d)-EAT graph

In this section, we construct ASD-antimagic labelings from (a, d)-EATLs of paths, stars, and cycles. In our constructions, an ascending sequence with particular properties is needed, and thus the following lemma.

Lemma 3.1. *Let t and q be two positive integers, where $\binom{t+1}{2} \leq q < \binom{t+2}{2}$. There exists an ascending sequence $\{b_i\}_{i=1}^{t}$ of length t where $\sum_{i=1}^{t} b_i = q$.*

Proof. If $q = \binom{t+1}{2}$, we have $q = \sum_{i=1}^{t} i$. If $\binom{t+1}{2} < q < \binom{t+2}{2}$, q can be written as $q = \sum_{i=1}^{t-1} i + (q - \binom{t}{2})$. Both cases give us the required ascending sequence. □

We also need to define a decomposition of an (a, d)-EAT graph such that the weights of its subgraphs form an ascending sequence. In this definition, we need the notion of an edge-induced subgraph of a graph. Suppose E' is a nonempty subset of $E(G)$. The subgraph of G whose vertex set is the set of end vertices of edges in E' and whose edge set is E' is called the *subgraph of G induced by E'*, denoted by $G[E']$ [5].

Definition 3.1. *Let G admit an (a, d)-EATL f and $b_1, b_2, \ldots, b_t$ be an ascending sequence of positive integers. An **ascending weight decomposition (AWD)** of G is a decomposition of G into subgraphs $H_1, H_2, \ldots H_t$, where $|E(H_i)| = b_i$, $1 \leq i \leq t$, as a result of the following steps:*

1. *Denote by e_i the edge with $w(e_i) = a + (i-1)d$, where $w(e)$ is the weight of an edge e under f.*
2. *Define $E(H_1) = \{e_m \mid 1 \leq m \leq b_1\}$ and, for $i \geq 2$, $E(H_i) = \{e_m \mid \sum_{j=1}^{i-1} b_j < m \leq \sum_{j=1}^{i-1} b_j + b_i\}$.*
3. *For $1 \leq i \leq t$, define H_i as a subgraph induced by the edge set $E(H_i)$, that is, $H_i = G[E(H_i)]$.*

Now, we are ready to construct super ASD-antimagic labelings for a super (a, d)-EAT graph G, where G is either the star $K_{1,n}$, the path P_n, or the cycle C_n through the following steps.

Algorithm 1 Super ASD-antimagic labeling for an (a, d)-EAT graph

1. Apply Lemma 3.1, to find an ascending sequence $\{b_i\}_{i=1}^{t}$ of size t, where $\binom{t+1}{2} \leq |E(G)| < \binom{t+2}{2}$ such that $\sum_{i=1}^{t} b_i = |E(G)|$.
2. Apply AWD to G and obtain t subgraphs $H_1, H_2, \ldots, H_t$, with $|E(H_i)| = b_i$.
3. Show that $w(H_i) < w(H_{i+1})$, for $1 \leq i \leq t-1$.

We shall utilize the following super (a, d)-EAT labelings for stars, paths, and cycles to obtain their super ASD-antimagic labelings.

Theorem 3.1. [19] *The star $K_{1,n}$ has a super (a, d)-EATL if and only if one of the following conditions is satisfied*

i) $d \in \{0, 1, 2\}$ *and* $n \geq 1$,

ii) $d = 3$ *and* $1 \leq n \leq 2$.

Theorem 3.2. [4] *The path P_n, $n \geq 2$, has a super (a, d)-EAT labeling if and only if $d \in \{0, 1, 2, 3\}$.*

Theorem 3.3. [3] *The cycle C_n has a super (a, d)-EATL if and only if one of the following conditions is satisfied*

i) $d \in \{0, 2\}$ *and* $n \geq 3$ *is odd,*

ii) $d = 1$ *and* $n \geq 3$.

The following super ASD-antimagic labelings for stars, paths, and cycles are constructed by applying the steps in Algorithm 1 to graphs with super (a, d)-EAT labelings in Theorems 3.1, 3.2, and 3.3.

Theorem 3.4. *For $n \geq 1$, the star $K_{1,n}$ is super ASD-antimagic.*

Proof. Let $V(K_{1,n}) = \{v, v_i \mid 1 \leq i \leq n\}$, where v is the vertex of degree n and v_is are leaves, while $E(K_{1,n}) = \{e_i \mid 1 \leq i \leq n\}$. The super (a, d)-EATL of $K_{1,n}$ f from the proof of Theorem 3.1 is defined as $f(v) = 1$, $f(v_i) = i + 1$, $1 \leq i \leq n$, and $f(e_i) = n + 1 + i$. Applying Step 2 in Algorithm 1, we obtain $H_i \cong K_{1,b_i}$, and so $w(H_i) < w(H_{i+1})$, for $1 \leq i \leq t - 1$. Therefore, f is also a super ASD-antimagic labeling of $K_{1,n}$. □

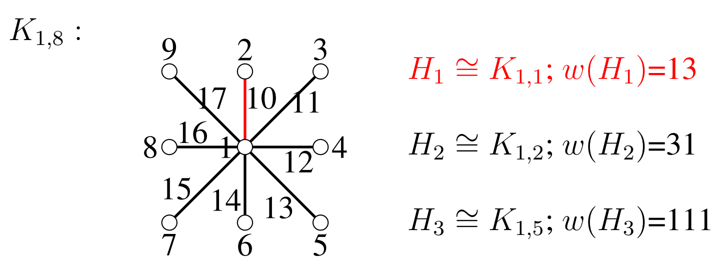

Figure 5. A super $(13, 2)$-EATL and a super ASD-antimagic labeling on $K_{1,8}$.

Figure 5 provides an example of a super $(13, 2)$-EATL of a star $K_{1,8}$. With the same labeling, $K_{1,8}$ is also super ASD-antimagic with ascending subgraphs $H_1 \cong K_{1,1}$, $H_2 \cong K_{1,2}$, and $H_3 \cong K_{1,5}$ and weights $w(H_1) = 13$, $w(H_2) = 31$, $w(H_3) = 111$, respectively.

Theorem 3.5. *For $n \geq 2$, the path P_n is super ASD-antimagic.*

Proof. Let $P_n : v_1 - e_1 - v_2 - e_2 - \cdots - e_{n-1} - v_n$. The super (a, d)-EATL f for P_n from the proof of Theorem 3.2 is defined as $f(v_i) = i$ and $f(e_i) = n + i$. Applying Step 2 in Algorithm 1, we obtain $H_i \cong P_{b_i+1}$. Thus, $w(H_i) < w(H_{i+1})$, for $1 \leq i \leq t - 1$. Therefore, f is also a super ASD-antimagic labeling of P_n. □

Figure 6 illustrates an example of a super $(13, 3)$-EATL of path P_9. With the same labeling, P_9 is also super ASD-antimagic with ascending subgraphs $H_1 \cong P_2$, $H_2 \cong P_3$, and $H_3 \cong P_6$ with weights $w(H_1) = 13$, $w(H_2) = 32$, $w(H_3) = 114$, respectively.

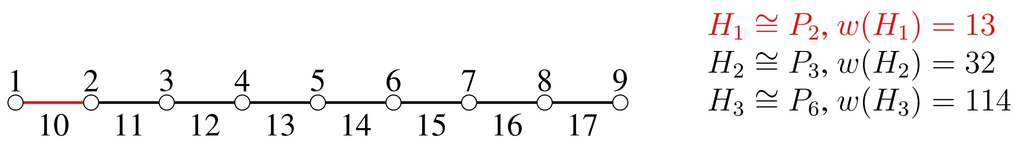

Figure 6. A super $(13, 3)$-EATL and a super ASD-antimagic labeling on P_9.

Theorem 3.6. *For $n \geq 3$, the cycle C_n is super ASD-antimagic.*

Proof. Let $C_n : v_1 - e_1 - v_2 - e_2 - \cdots - e_{n-1} - v_n - e_n - v_1$. The super (a, d)-EATL f for C_n from the proof of Theorem 3.3 is defined as $f(v_i) = i$, and $f(e_i) = 2n + 1 - i$. Applying Step 2 in Algorithm 1, we obtain $H_i \cong P_{b_i+1}$, and so $w(H_i) < w(H_{i+1})$, for $1 \leq i \leq t - 1$. Therefore, f is also a super ASD-antimagic labeling of C_n. □

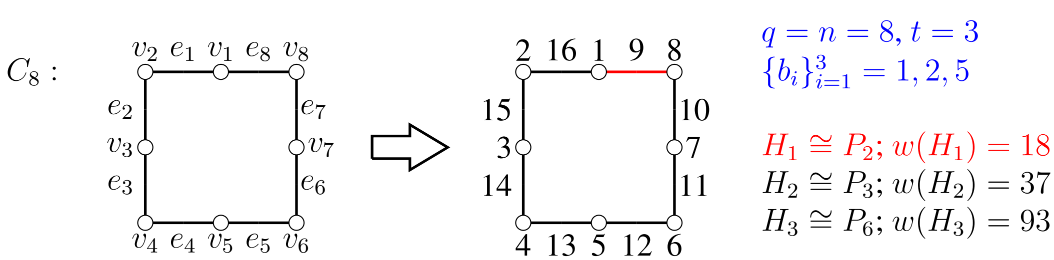

Figure 7. A super $(18, 1)$-EATL and a super ASD-antimagic labeling on C_8.

Figure 7 gives an example of a super $(18, 1)$-EATL on C_8. This labeling is also a super ASD-antimagic with $H_1 \cong P_2$, $H_2 \cong P_3$, and $H_3 \cong P_6$.

Note that for stars (Theorem 3.4), we can use any super (a, d)-EATL of stars to obtain a super ASD-antimagic labeling, since the subgraphs obtained from AWD is obviously ASD. However, for paths (Theorem 3.5) and cycles (Theorem 3.6), we have to use particular (a, d)-EATLs of paths and cycles in order to ensure that the subgraphs obtained from AWD are ASD. In general, we can not apply AWD to arbitrary (a, d)-EATL of a graph to obtain a super ASD-antimagic labeling since it may not result in an ASD. As an example, consider Figure 8, where we have a $(27, 3)$-EATL of a caterpillar T. Applying AWD to T, we obtain subgraphs $H_1, H_2, \ldots, H_6$ which are not ASD, since H_4 is not isomorphic to a proper subgraph of H_5.

Hence, we ask whether there exist methods other than AWD to obtain a super ASD-antimagic labeling from a super (a, d)-EATL and propose the following open question.

Problem 1. *Does a super (a, d)-EATL of a graph G always result in a super ASD-antimagic labeling of G?*

We conclude this section by considering ASD-antimagic labelings of complete graphs. Alavi *et. al.* [2] proved that for a complete graph K_n, there is a natural ASD into stars, that is, $H_i \cong K_{1,i}$, for $1 \leq i \leq n - 1$.

Theorem 3.7. *For $n \geq 2$, the complete graph K_n is super ASD-antimagic.*

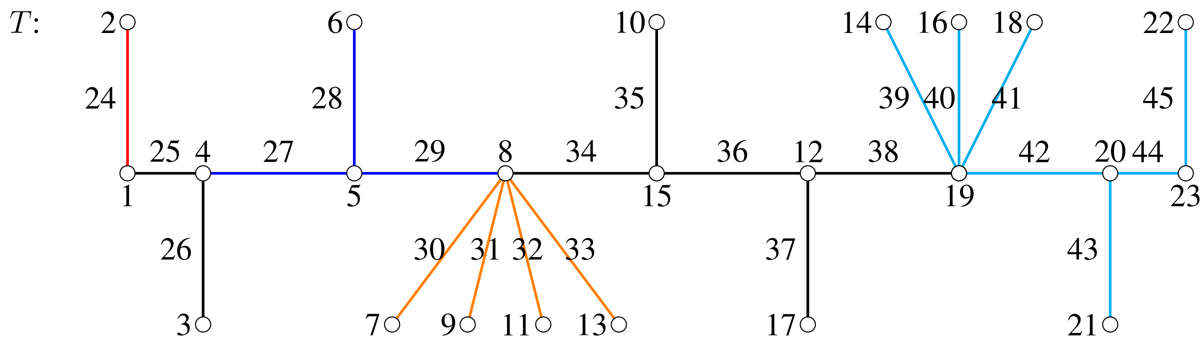

Figure 8. A super $(27, 3)$-EATL on a caterpillar that is not ASD-antimagic labeling.

Proof. Decompose K_n into stars, that is $H_i \cong K_{1,i}$ for $1 \leq i \leq t$. Label the vertices using $\{1, 2, \ldots, n\}$. For $i = 1$, we set the label for H_1 as $f(H_1) = \{1, 2, n+1\}$, so we have $w(H_1) = n + 4$. Next, for $2 \leq i \leq t$, we set the label for H_i as $f(H_i) = \{r \mid 1 \leq r \leq i+1\} \cup \{s \mid n+1+\binom{i}{2} \leq s \leq n+\binom{i+1}{2}\}$. Hence, we obtain $w(H_i) = \binom{i+2}{2} + i(n+1) + \binom{i}{2}(i+1)$. Therefore, K_n admits a super ASD-antimagic labeling. □

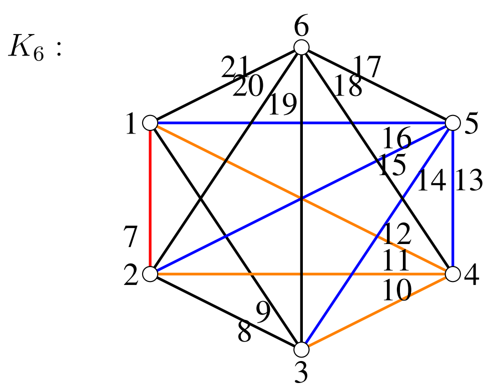

Figure 9. A super ASD-antimagic labeling on K_6.

Finally, we note that if a graph admits an ASD, it is reasonable to think that it also admits an ASD-antimagic labeling. However, a general proof might be difficult to obtain. Therefore, we boldly propose the following.

Conjecture 3.1. *All ASD graphs are ASD-antimagic.*

Recall Conjecture 1.1, where Alavi et al. conjectured that "All graphs of positive size are ASD". Combining with Conjecture 3.1, we obtain the following.

Conjecture 3.2. *All graphs of positive size are ASD-antimagic.*

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