Abstract

An $H$-magic labeling in an $H$-decomposable graph $G$ is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that for every copy $H$ in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ is constant. The function $f$ is said to be $H$-$E$-super magic if $f(E(G)) = \{1, 2, \ldots, q\}$. In this paper, we study some basic properties of $m$-factor-$E$-super magic labeling and we provide a necessary and sufficient condition for an even regular graph to be $2$-factor-$E$-super magic decomposable. For this purpose, we use Petersen’s theorem and magic squares.

Keywords: $H$-decomposable graph; $H$-$E$-super magic labeling; $2$-factor-$E$-super magic decomposable graph

Mathematics Subject Classification : 05C78

1. Introduction

In this paper, we consider only finite and simple undirected graphs. The vertex and edge sets of a graph $G$ are denoted by $V(G)$ and $E(G)$ respectively and we let $|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations, we follow [3, 4]. A labeling of a graph $G$ is mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labelings have been studied and an excellent survey of graph labeling can be found in [5].
The notion of an $E$-super vertex magic labeling was introduced by Swaminathan and Jeyanthi [15] as in the name of super vertex magic labeling and it was renamed as $E$-super vertex magic labeling by Marimuthu and Balakrishnan in [10]. A vertex magic total labeling is a bijection $f$ from $V(G) \cup E(G)$ to the integers $1, 2, 3, \ldots, p + q$ with the property that for every $u \in V(G)$, $f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant $k$. Such a labeling is $E$-super if $f(E(G)) = \{1, 2, 3, \ldots, q\}$. A graph $G$ is called $E$-super vertex magic if it admits an $E$-super vertex magic labeling. There are many graphs that have been proved to be an $E$-super vertex magic graph; see for instance [10, 15, 16]. In [10], Marimuthu and Balakrishnan proved that if a graph $G$ of odd order can be decomposed into two Hamilton cycles, then $G$ is an $E$-super vertex magic graph. The results of the article [10] can be found in [11]. In [17], Tao-Ming Wang and Guang-Hui Zhang gave the generalization of some results stated in [10] using 2-factors.

A covering of $G$ is a family of subgraphs $H_1, H_2, \ldots, H_h$ such that each edge of $E(G)$ belongs to at least one of the subgraphs $H_i, 1 \leq i \leq h$. Then, it is said that $G$ admits an $(H_1, H_2, \ldots, H_h)$ covering. If every $H_i$ is isomorphic to a given graph $H$, then $G$ admits an $H$-covering. A family of subgraphs $H_1, H_2, \ldots, H_h$ of $G$ is an $H$-decomposition of $G$ if all the subgraphs are isomorphic to a graph $H, E(H_i) \cap E(H_j) = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{h} E(H_i) = E(G)$. In this case, we write $G = H_1 \oplus H_2 \oplus \cdots \oplus H_h$ and $G$ is said to be $H$-decomposable. Suppose $G$ is $H$-decomposable. A total labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ is called an $H$-magic labeling of $G$ if there exists a positive integer $k$ (called magic constant) such that for every copy $H$ in the decomposition, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k$. A graph $G$ that admits such a labeling is called an $H$-magic decomposable graph. An $H$-magic labeling $f$ is called and an $H$-$E$-super magic labeling if $f(E(G)) = \{1, 2, \ldots, q\}$. A graph that admits an $H$-$E$-super magic labeling is called an $H$-$E$-super magic decomposable graph. The sum of all vertex and edge labels on $H$ is denoted by $\sum f(H)$.

The notion of $H$-super magic labeling was first studied by Gutiérrez and Lladó [6] in 2005. They proved that some classes of connected graphs are $H$-super magic. In 2007, Lladó and Moragas [8] studied the cycle-magic and cyclic-super magic behavior of several classes of connected graphs. They gave several families of $C_r$-magic graphs for each $r \geq 3$. In 2010, Ngurah, Salman and Susilowati [13] studied the cycle-super magic labeling of chain graphs, fans, triangle ladders, graphs obtained by joining a star $K_{1,n}$ with one isolated vertex, grids and books. Maryati et al. [12] studied the $H$-super magic labeling of some graphs obtained from $k$ isomorphic copies of a connected graph $H$. In 2012, Roswitha and Baskoro [9] studied the $H$-super magic labeling for some classes of trees such as a double star, a caterpillar, a firecracker and a banana tree. In 2013, Kojima [18] studied the $C_4$-super magic labeling of the Cartesian product of paths and graphs. In 2012, Inayah et al. [7] studied magic and antimagic $H$-decompositions and Liang [19] studied cycle-super magic decompositions of complete multipartite graphs. In these above results, they call an $H$-magic labeling as an $H$-super magic if the smallest labels are assigned to the vertices. Here, we call an $H$-magic labeling as an $H$-$E$-super magic if the smallest labels are assigned to the edges. In many of the results about $H$-magic graphs, the host graph $G$ is required to be $H$-decomposable. If $H \cong K_2$, then an $H$-magic graph is an edge magic graph. The definition of
an $H$-magic decomposition is suggested by this observation. Also it is notable that the notions of super edge magic and $E$-super edge magic are the same [11].

Any spanning subgraph of a graph $G$ is referred to as a factor of $G$. An $m$-regular factor is called an $m$-factor. A graph $G$ is said to be factorable into the factors $G_1, G_2, \ldots, G_h$ if these factors are pairwise edge-disjoint and $\bigcup_{i=1}^{h} E(G_i) = E(G)$. If $G$ is factored into $G_1, G_2, \ldots, G_h$, then we represent this by $G = G_1 \oplus G_2 \oplus \cdots \oplus G_h$, which is called a factorization of $G$. It is nothing but the factor-decomposition. If there exists a factor-decomposition of a graph $G$ such that each factor is a $m$-factor, then $G$ is $m$-factor-decomposable. If $G$ is a $m$-factor-decomposable graph, then necessarily $G$ is $r$-regular for some integer $r$ that is a multiple of $m$. Of course, for a graph to be $2$-factor-decomposable, it is necessary that it be $2r$-regular for some integer $r \geq 1$. Petersen [14] showed that this obvious necessary condition is sufficient as well.

**Theorem 1.1.** [14] Every $2r$-regular graph has a $2k$-factor for every integer $k, 0 < k < r$.

Magic squares are among the more popular mathematical recreations. A classical reference on magic squares is [1], while one of the better recent book is [2]. A magic square of side $n$ is an $n \times n$ array whose entries are an arrangement of integers $\{1, 2, \ldots, n^2\}$ in which all elements in any row, any column or either main diagonal or back-diagonal, add to the same sum. Furthermore, we denote this sum as magic number (MN) and also we observe that the value of the magic number is $MN = \frac{1}{2}n(n^2 + 1)$.

In this paper, first we study the elementary properties of $m$-factor-$E$-super magic graphs and then we present a necessary and sufficient condition for an even regular graph to be $2$-factor-$E$-super magic decomposable. To prove these results, we use Petersen’s theorem and magic squares.

2. $m$-factor-$E$-Super magic graphs

This section will explore the basic properties of $m$-factor-$E$-super magic graphs.

**Lemma 2.1.** If a non-trivial $m$-factor-decomposable graph $G$ is $m$-factor-$E$-super magic decomposable, then the magic constant $k$ is $\frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$, where $h$ is the number of $m$-factors of $G$.

**Proof.** Let $f$ be an $m$-factor-$E$-super magic labeling of a graph $G$ with the magic constant $k$. Then $f(E(G)) = \{1, 2, \ldots, q\}$, $f(V(G)) = \{q+1, q+2, \ldots, q+p\}$, and $k = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ for every factor $G'$ in the decomposition of $G$. Then,

$$hk = \sum_{e \in E(G)} f(e) + h \sum_{v \in V(G)} f(v)$$

$$= [1 + 2 + \cdots + q] + h[q + 1 + q + 2 + \cdots + q + p]$$

$$= \frac{q(q+1)}{2} + h \left[ pq + \frac{p(p+1)}{2} \right]$$

Thus, $k = \frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$. \qed
If $G$ is an $m$-factor-decomposable graph and $G$ possesses an $m$-factor-$E$-super magic labeling, then we can easily find the sum of the vertex labels (denoted by $k_v$) in each factor and are the same. This gives the following result.

**Lemma 2.2.** If a non-trivial $m$-factor-decomposable graph $G$ is $m$-factor-$E$-super magic decomposable, then the sum of the edge labels, denoted by $k_e$, is a constant and it is given by $k_e = \frac{q(q+1)}{2h}$, where $h$ is the number of $m$-factors of $G$.

**Proof.** Suppose that $G$ is $m$-factor-decomposable and $G$ has an $m$-factor-$E$-super magic labeling $f$. Then, by Lemma 2.1, the magic constant $k$ is given by $k = \frac{q(q+1)}{2h} + pq + \frac{p(p+1)}{2}$ for every $m$-factor $G'$ in the decomposition of $G$. Since $G$ is $m$-factor-decomposable, every $m$-factor $G'$ in the decomposition of $G$ is a spanning subgraph of $G$. It follows that $k_v$ is constant for every $m$-factor $G'$ of $G$. Since $k = k_v + k_e$, then $k_e$ must be a constant. Also, $hk_e = \sum_{e \in E(G)} f(e) = 1 + 2 + \cdots + q = \frac{q(q+1)}{2}$ and hence $k_e = \frac{q(q+1)}{2h}$. $\square$

In addition, the following lemma gives a necessary and sufficient condition for an $m$-factor-decomposable graph to be $m$-factor-$E$-super magic decomposable. This lemma is helpful in deciding whether a particular graph has an $m$-factor-$E$-super magic labeling.

**Lemma 2.3.** Let $G$ be a $m$-factor-decomposable graph and let $g$ be a bijection from $E(G)$ onto $\{1, 2, \ldots, q\}$. Then $g$ can be extended to an $m$-factor-$E$-super magic labeling of $G$ if and only if $k_e = \sum_{e \in E(G)} g(e)$ is constant for every $m$-factor $G'$ in the decomposition of $G$.

**Proof.** Suppose that $G$ can be decomposed into some $m$-factors. Assume that $k_e = \sum_{e \in E(G)} f(e)$ is constant for every $m$-factor $G'$ in the decomposition of $G$. Define $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ as $f(uv) = g(uv)$ for $uv \in E(G)$ and $f(v_i) = q + i$ for all $i = 1, 2, \ldots, p$. Then $f(E(G)) = \{1, 2, \ldots, q\}$ and $f(V(G)) = \{q + 1, q + 2, \ldots, q + p\}$. Since every $m$-factor $G'$ of $G$ is a spanning subgraph of $G$, $k_v = \sum_{v \in V(G')} f(v)$ is constant for every $m$-factor $G'$ in the decomposition of $G$. Therefore $k_v + k_e = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ is a constant for every $m$-factor $G'$ in the decomposition of $G$. Thus, we have that $f$ is an $m$-factor-$E$-super magic labeling of $G$. Suppose $g$ can be extended to a $m$-factor-$E$-super magic labeling $f$ of $G$ with a magic constant $k$. Then, $k = \sum_{v \in V(G')} f(v) + \sum_{e \in E(G')} f(e)$ for every $m$-factor $G'$ in the decomposition of $G$. Since $G$ is $m$-factor-decomposable, $k_v = \sum_{v \in V(G')} f(v)$ is constant and it follows that $k_e = \sum_{e \in E(G')} f(e)$ is also a constant for every $m$-factor $G'$ in the decomposition of $G$. $\square$

### 3. Necessary and sufficient condition

Based on the lemmas stated in the previous section, the problem of finding an $m$-factor-$E$-super magic labeling of $m$-factor-decomposable graphs is difficult. So, we restrict our attention to 2-factor-decomposable graphs. In this section, we discuss the 2-factor-$E$-super magic labeling of
2-factor-decomposable graphs. The following theorem is useful in finding classes of graphs that are not 2-factor-$E$-super magic.

**Theorem 3.1.** An even regular graph $G$ of odd order is not 2-factor-$E$-super magic decomposable, when the number of factors $h$ is even.

**Proof.** Let $G$ be an even regular graph of odd order. Then by Petersen’s theorem, $G$ is 2-factor-decomposable. Suppose $G$ is a 2-factor-$E$-super magic decomposable graph. Then $G$ has a 2-factor-$E$-super magic labeling. By Lemma 2.2, we have $k_e = \frac{q(q+1)}{2h}$. Since $G$ is 2-factor-decomposable with $h$ 2-factors, $q = ph$. Therefore, $k_e = \frac{ph(ph+1)}{2h} = \frac{p(p+1)}{2}$. It is given that $G$ is of odd order. We take $p = 2t+1$. Therefore,

$$k_e = \frac{(2t+1)[(2t+1)h+1]}{2} = 2t^2h + 2th + t + \frac{h+1}{2},$$

which is an integer only if $h$ is odd and hence $G$ is not a 2-factor-$E$-super magic decomposable if $h$ is even. \qed

The following theorem provides a necessary and sufficient condition for an even regular graph $G$ of odd order to be 2-factor-$E$-super magic decomposable.

**Theorem 3.2.** An even regular graph $G$ of odd order is 2-factor-$E$-super magic decomposable if and only if $h$ is odd, where $h$ is the number of 2-factors of $G$.

**Proof.** Let $G$ be an even regular graph of odd order $p$. If $h$ is even, by Theorem 3.1, $G$ is not 2-factor-$E$-super magic. Suppose that $h$ is odd. Then, by Petersen’s theorem, $G$ can be decomposed into 2-factors which is the sum say $G = F_1 \oplus F_2 \oplus \cdots \oplus F_h$ where $F_i$ is a 2-factor for each $i, 1 \leq i \leq h$. Now, the edges of $G$ can be labeled as shown in Table 1.

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<td>$(p-1)h + 2$</td>
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Table 1. The edge label of an odd order 2-factor-decomposable graph $G$ if $h$ is odd.

From Table 1, the sum of the edge labels at the 2-factor $F_1$ in the decomposition is calculated as follows:
\[
\sum_{e \in E(F_1)} f(e) = MN + h^2 + 1 + h^2 + 2h + h^2 + 2h + 1 + h^2 + 4h + h^2 + 4h + 1
\]
\[+ \cdots + h^2 + (p - h - 2)h + 1 + h^2 + (p - h)h,\]
where \(MN\) = magic number of \(h \times h\) magic square
\[= MN + (p - h)h^2 + 2(2h + 4h + \cdots + (p - h - 2)h)
\[+ \left[1 + 1 + \cdots + 1\right] + (p - h)h\]
\[= MN + \frac{(p-h-2)}{2} \left(\frac{(p-h-2)}{2} + 1\right) + \frac{p-h}{2} + (p-h)h\]
\[= MN + (p - h)h^2 + 4h \left(\frac{p-h}{2}\right) + \frac{p-h}{2} + (p-h)h\]
\[= \frac{h^3}{2} + \frac{h}{2} + ph^2 - h^3 + \frac{p^2h}{2} - \frac{p^2h}{2} - \frac{ph^2}{2} + \frac{h^3}{2} - \frac{2ph}{2}
\[+ \frac{2h^2}{2} + \frac{p}{2} - \frac{h}{2} + ph - h^2
\[= \frac{p^2h + p}{2}.
\]
In a similar way, we can calculate that the sum of the edge labels at each 2-factor in the decomposition is the constant \(k_e = \frac{p^2h + p}{2}\). Then, by Lemma 2.3, this labeling can be extended to a 2-factor-\(E\)-super magic labeling.

Examples 1 and 2 illustrate Theorem 3.2.

![Figure 1. The complete graph \(K_7\) is 2-factor-\(E\)-super magic.](image-url)
Example 1. The complete graph $K_7$ is decomposed into three 2-factors, namely $K_7 = F_1 \oplus F_2 \oplus F_3$. The edges of each factor-decomposition of Figure 2 are labeled as shown in the Table 2.

Table 2. A 2-factor-\(E\)-super magic labeling of $K_7$.

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Example 2. The complete graph $K_{11}$ can be decomposed into five 2-factors say $K_{11} = F_1 \oplus F_2 \oplus F_3 \oplus F_4 \oplus F_5$. The edge labels of each factor of $K_{11}$ are shown above in Table 3. In Table 3, the sum of the edge labels at each factor is $k_e = 308$. Then, by using Lemma 2.3, we extend this edge labeling to a 2-factor-\(E\)-super magic labeling.

Theorem 3.3. An even regular graph \(G\) of even order is 2-factor-\(E\)-super magic decomposable.

Proof. Let \(G\) be an even regular graph of even order \(p\). By Petersen’s theorem \(G\) can be decom-
posed into 2-factors which is the sum, say $G = F_1 \oplus F_2 \oplus F_3 \oplus \cdots \oplus F_h$, where $F_i$ is a 2-factor for each $i, 1 \leq i \leq h$. Now, the edges of $G$ can be labeled as shown in the Table 4.

From Table 4, the sum of the edge labels at the 2-factor $F_1$ in the decomposition is calculated as follows:

$$
\sum_{e \in E(F_1)} f(e) = 1 + 2h + 2h + 1 + 4h + 4h + 1 + \cdots + ph \\
= 1 + 2[2h + 4h + \cdots + (p - 2)h] + \left\{ 1 + 1 + \cdots + 1 \right\} + ph \\
= 4h \left[ 1 + 2 + \cdots + \frac{p - 2}{2} \right] + \frac{p}{2} + ph \\
= \frac{p^2 h}{2} - ph + \frac{p}{2} + ph = \frac{p^2 h + p}{2}.
$$

In similar way, we can calculate that the sum of the edge labels at each factor-decomposition is the constant $k_e = \frac{p^2 h + p}{2}$. Then, by Lemma 2.3, this labeling can be extended to a 2-factor-$E$-
Examples 3 and 4 illustrate Theorem 3.3.

**Example 3.** The following graph $G$ can be decomposed into three 2-factors say $G = F_1 \oplus F_2 \oplus F_3$. Note that one of the factors is disconnected.

![Figure 3. The graph $G$ is 2-factor-$E$-super magic.](image)

![Figure 4. The 2-factor-decomposition of the graph $G$.](image)

The edges of each factor-decomposition of Figure 4 are labeled as shown in Table 5. 
In Table 5, the sum of the edge labels at each factor-decomposition is $k_e = 100$. Then, by Lemma 2.3, we extend this edge labeling to a 2-factor-$E$-super magic labeling.
$H$-E-Super magic decomposition of graphs  |  S. P. Subbiah and J. Pandimadevi

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Table 5. 2-factor-E-super magic labeling of $G$.

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Table 6. 2-factor-E-super magic labeling of $G$.

Figure 5. The graph $G$ is 2-factor-E-super magic decomposable.
Example 4. The graph $G$ shown in Figure 5 can be decomposed into four 2-factors say $G = F_1 \oplus F_2 \oplus F_3 \oplus F_4$.

![Figure 6. The 2-factor-decomposition of the graph $G$.](image)

The edges of each factors of Figure 6 are labeled as shown in Table 6. In Table 6, the sum of the edge labels at each factor-decomposition is $k_e = 205$. Then, by using Lemma 2.3, we extend this edge labeling to a 2-factor-$E$-super magic labeling.

4. Conclusion

In this paper, we have given a complete characterization of 2-factor-$E$-super magic decomposable graphs. Furthermore, we can find some examples of 1-factor-$E$-super magic decomposable graphs (see Figures 7 and 8). The complete graph $K_6$ can be decomposed into five 1-factors say $K_6 = F_1 \oplus F_2 \oplus F_3 \oplus F_4 \oplus F_5$.

In Figure 7, the sum of the edge labels at each factor-decomposition is $k_e = 24$. Since, every 1-factor-decomposition is a spanning subgraph of $K_6$, then sum of the labels on edges and vertices of each factor is $k_v + k_e$ is constant and hence $K_6$ is 1-factor-$E$-super magic decomposable.
Thus, we conclude this paper with the following open problem.

**Open Problem 1.** Characterize all $m$-factor-$E$-super magic decomposable graphs, $m \neq 2$.

**Acknowledgement**

The authors are thankful to the anonymous referees for their helpful suggestions.
References


